

$$\text{أما } Z=\omega \Rightarrow \frac{1+3\omega^{10}+3\omega^{11}}{1-3\omega^7-3\omega^8}$$

$$= \frac{1+3\omega+3\omega^2}{1-3\omega-3\omega^2} = \frac{1+3(\omega+\omega^2)}{1-3(\omega+\omega^2)} = \frac{1-3}{1+3}$$

$$= -\frac{2}{4} = \left(-\frac{1}{2}\right)$$

$$\text{أما } Z=\omega^2 \Rightarrow \frac{1+3(\omega^2)^{10}+3(\omega^2)^{11}}{1-3(\omega^2)^7-3(\omega^2)^8}$$

$$= \frac{1+3\omega^{20}+3\omega^{22}}{1-3\omega^{14}-3\omega^{16}} = \frac{1+3\omega^2+3\omega}{1-3\omega^2-3\omega}$$

$$= \frac{1+3(\omega^2+\omega)}{1-3(\omega^2+\omega)} = \frac{1-3}{1+3} = -\frac{2}{4} = \left(-\frac{1}{2}\right)$$

مزاوي 2018 المصدي

ن/ اذا كان $Z^2+Z+1=0$ نجد

$$\frac{1+3Z^{10}+3Z^{11}}{1-3Z^7-3Z^8}$$

$$Z^2+Z+1=0 \quad (a=1) \quad (b=1) \quad (c=1)$$

$$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$Z = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$S = \left\{ -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right\}$$

$$S = \{\omega, \omega^2\}$$

الجزور التكعيبية للواحد الصحيح «الأوميا»

$$Z^3 = 1$$

$$Z^3 - 1 = 0$$

$$(Z-1)(Z^2+Z+1) = 0$$

$$\text{أما } Z=1 \text{ أو } (Z^2+Z+1=0) \quad (a=1) \quad (b=1) \quad (c=1)$$

$$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$Z = \frac{-1 \pm \sqrt{-3}}{2} \Rightarrow Z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$S = \left\{ 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right\}$$

$$S = \{1, \omega, \omega^2\}$$

$$S = \{1, \omega^2, \omega\}$$

وزارة 2018 الدور الثاني

$$(x+yi)(2+i) = \frac{1}{(1+\omega)^2} + \frac{1}{(1+\omega^2)^2}; x, y \in \mathbb{R} \text{ من جد قيمة}$$

$$(x+yi)(2+i) = \frac{1}{(-\omega^2)^2} + \frac{1}{(-\omega)^2}$$

$$(x+yi)(2+i) = \frac{1}{\omega^4} + \frac{1}{\omega^2}$$

$$(x+yi)(2+i) = \frac{\omega^8}{\omega} + \frac{\omega^8}{\omega^2}$$

$$(x+yi)(2+i) = \omega^7 + \omega \Rightarrow \frac{(x+yi)(2+i)}{2+i} = \frac{-1}{2+i} \cdot \frac{2-i}{2-i}$$

$$x+yi = \frac{-2+i}{5}$$

$$x+yi = -\frac{2}{5} + \frac{1}{5}i \quad x = -\frac{2}{5} \quad y = \frac{1}{5}$$

وزارة 2015 الدور الأول

$$(x+yi)(1-\sqrt{3}i) = -2\omega - 2\omega^2; x, y \in \mathbb{R} \text{ من جد قيمة}$$

$$(x+yi)(1-\sqrt{3}i) = -2(\omega + \omega^2)$$

$$\frac{(x+yi)(1-\sqrt{3}i)}{(1-\sqrt{3}i)} = \frac{2}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$$

$$x+yi = \frac{2+2\sqrt{3}i}{4}$$

$$x+yi = \frac{2}{4} + \frac{2\sqrt{3}}{4}i$$

$$x+yi = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$x = \frac{1}{2}$$

$$y = \frac{\sqrt{3}}{2}$$

وزارة 2024 الدرر الثالث

من إذا كان $x, y \in \mathbb{R}$ عددان مترافقان فقيمة $x+yi$

$$\frac{7-i\omega-i\omega^2}{3+i\omega+i\omega^2} = \frac{7-i(\omega+\omega^2)}{3+i(\omega+\omega^2)} = \frac{7-i}{3-i}$$

$$x+yi = \frac{20-10i}{3-i}$$

$$x+yi = \frac{60-20i-30i-10}{10}$$

$$x+yi = \frac{50}{10} - \frac{50i}{10}$$

$$x+yi = 5-5i$$

$$x=5$$

$$y=-5$$

$$\frac{x+yi}{3-i} = \frac{7-i}{3-i}$$

$$\frac{x+yi}{3-i} = \frac{7-i}{3-i}$$

$$(3+i)(x+yi) = (3-i)(7-i)$$

$$(3+i)(x+yi) = 21-3i-7i-1$$

$$\frac{(3+i)(x+yi)}{3+i} = \frac{20-10i}{3+i}$$

وزارة 2017 الدرر الأول

من إذا كان $x, y \in \mathbb{R}$ فقيمة $x+yi$

$$x+yi = (\sqrt{\omega+\omega^7} + \sqrt{\omega+\omega^{38}})^2 - \frac{3+i}{1+i}$$

$$x+yi = (\sqrt{\omega+\omega^2} + \sqrt{\omega+\omega^2})^2 - \left(\frac{3+i}{1+i} \cdot \frac{1-i}{1-i} \right)$$

$$x+yi = (\sqrt{1} + \sqrt{1})^2 - \left(\frac{3-3i+i+1}{2} \right)$$

$$x+yi = (i+i)^2 - \left(\frac{4}{2} - \frac{2i}{2} \right)$$

$$x+yi = (2i)^2 - (2-i)$$

$$x+yi = -4-2+i$$

$$x+yi = -6+i$$

$$x=-6, y=1$$

وزاري 2025 الدور الأول

من أكتب العدد Z بالصيغة العادية ثم جد

الجذور التربيعية للعدد: $Z = (\sqrt{9} + \sqrt{2}w)^6 + \sqrt{36}$

$$x^2 - \frac{9}{x^2} = 8 \Rightarrow x^4 - 9 = 8x^2$$

$$x^4 - 9 = 8x^2$$

$$x^4 - 8x^2 - 9 = 0$$

$$(x^2 - 9)(x^2 + 1) = 0$$

$$x^2 = 9 \Rightarrow x = \pm 3$$

$$(x=3) (y=1)$$

$$(x=-3) (y=-1)$$

$$C_1 = 3+i \quad C_2 = -3-i$$

$$Z = [\sqrt{2}(1+w)]^6 + 6i$$

$$Z = (\sqrt{2}w)^6 + 6i$$

$$Z = 8w^3 + 6i$$

$$Z = 8 + 6i$$

$$(x+yi)^2 = 8+6i$$

$$x^2 - y^2 + 2xyi = 8+6i$$

$$x^2 - y^2 = 8 \quad (1) \quad \frac{2xy}{2x} = \frac{6}{2x}$$

$$y = \frac{3}{x} \quad (2)$$

وزاري 2022 الدور الثاني

من جد الجزران التربيعيان للعدد المركب $(w^2 + w) + 4i^7 + 4w^6$

$$x^4 - 3x^2 - 4 = 0$$

$$(x^2 - 4)(x^2 + 1) = 0$$

$$x^2 = 4 \Rightarrow x = \pm 2$$

$$(x=2) (y=-1)$$

$$(x=-2) (y=1)$$

$$C_1 = 2-i$$

$$C_2 = -2+i$$

$$4 - 4i - 1 = 3 - 4i$$

$$(x+yi)^2 = 3-4i$$

$$x^2 - y^2 + 2xyi = 3-4i$$

$$x^2 - y^2 = 3 \quad (1) \quad \frac{2xy}{2x} = \frac{-4}{2x}$$

$$y = -\frac{2}{x} \quad (2)$$

$$x^2 - \left(-\frac{2}{x}\right)^2 = 3$$

$$x^2 - \frac{4}{x^2} = 3 \Rightarrow x^4 - 4 = 3x^2$$

$$x^4 - 4 = 3x^2$$