

وزارة 2003 الدور الثاني

$$\begin{aligned}
 & \frac{1}{3+5\omega+4\omega^2} + \frac{1}{3+4\omega+5\omega^2} \\
 &= \frac{1}{3+\omega+4\omega+4\omega^2} + \frac{1}{3+4(\omega+4\omega^2+\omega^2)} \\
 &= \frac{1}{3+\omega+4(\omega+\omega^2)} + \frac{1}{3+4(\omega+\omega^2)+\omega^2} \\
 &= \frac{1}{3+\omega-4} + \frac{1}{3-4+\omega^2} \\
 &= \frac{1}{(\omega-1)} + \frac{1}{(\omega^2-1)} \\
 &= \frac{(\omega^2-1)+(\omega-1)}{(\omega-1)(\omega^2-1)} = \frac{\omega^2+\omega-2}{\omega^3-\omega-\omega^2+1} \\
 &= \frac{-1-2}{1+1+1} \\
 &= -\frac{3}{3} = \boxed{-1}
 \end{aligned}$$

وزارة 2017 الدور الثاني

$$\begin{aligned}
 & \left(3\omega^3 + \frac{5}{\omega^5} + \frac{4}{\omega^4}\right)^6 \\
 &= \left(3 + \frac{5\omega^3}{\omega^2} + \frac{4\omega^3}{\omega}\right)^6 = \left(3 + 5\omega + 4\omega^3\right)^6 \\
 &= \left(3 + \omega + 4\omega + 4\omega^3\right)^6 \\
 &= \left[3 + \omega + 4(\omega + \omega^3)\right]^6 \\
 &= (3 + \omega - 4)^6 = (\omega - 1)^6 = [(\omega - 1)^2]^3 \\
 &= (\omega^2 - 2\omega + 1)^3 \\
 &= (\omega^2 + 1 - 2\omega)^3 \\
 &= (-\omega - 2\omega)^3 \\
 &= (-3\omega)^3 = -27\omega^3 = \boxed{-27}
 \end{aligned}$$

وزارة 2003 الدور الأول

$$\begin{aligned}
 & \left(2 + 3\omega^2 + \omega\right)^2 \\
 &= (2 + 2\omega^2 + \omega^2 + \omega)^2 \\
 &= [2(1 + \omega^2) - 1]^2 \\
 &= (-2\omega - 1)^2 \\
 &= 4\omega^2 + 4\omega + 1 \\
 &= 4(\omega^2 + \omega) + 1 \\
 &= -4 + 1 \\
 &= \boxed{-3}
 \end{aligned}$$

وزارة 2019 الدور الأول

$$\left[\frac{1}{1+i} - \frac{1}{1-i} \right]^{100} = \left[\frac{2+3w}{2w^2+3} + \frac{4w^2+1}{4+w} \right]^{200} \quad \text{برأيت أن:}$$

$$R-h-S = \left(\frac{2w^3+3w}{2w^2+3} + \frac{4w^2+w^3}{4+w} \right)^{200} \\ = \left(\frac{w(2w^2+3)}{(2w^2+3)} + \frac{w^2(4+w)}{(4+w)} \right)^{200} = (w+w^2)^{200} = (-1)^{200} = 1$$

$$h-h-S = \left[\frac{1}{1+i} \cdot \frac{1-i}{1-i} - \frac{1}{1-i} \cdot \frac{1+i}{1+i} \right]^{100} \\ = \left[\frac{(1-i)}{2} - \frac{(1+i)}{2} \right]^{100} = \left(\frac{1-i-1-i}{2} \right)^{100} = \left(\frac{-2i}{2} \right)^{100} = (-i)^{100} = 1 \\ h-h-S = R-h-S$$

وزارة 2014 الدور الثاني

$$\left(\frac{5w^2i-1}{5+iw} \right)^6 = -1 \quad \text{برأيت أن:}$$

$$h-h-S = \left(\frac{5w^2i-1}{5+iw} \right)^6 \\ = \left[\frac{w^2i(5+iw)}{(5+iw)} \right]^6$$

$$= (w^2i)^6 \\ = w^{12} i^6 \quad \begin{matrix} 6 & 4(1) & 2 \\ i^6 = i^4 \cdot i^2 = -1 \end{matrix} \\ = (1)(-1) \\ = -1 = R-h-S$$

$$\left(\frac{7+3w}{7w^2+3} \right)^3 = 1 \quad \text{برأيت أن:}$$

$$h-h-S = \left(\frac{7w^3+3w}{7w^2+3} \right)^3 \\ = \left(\frac{w(7w^2+3)}{(7w^2+3)} \right)^3$$

$$= w^3 \\ = 1 \\ = R-h-S$$

وزارة 2023 الدور الأول المراجعة التمهيدية.

$$\left[\frac{a+b}{a+(2a+b)a+aw^2} \right]^6 = 1 \quad \text{بـ أثبت أن:}$$

$$h-h-S = \left[\frac{a+b}{a+aw^2+2aw+bw} \right]^6$$

$$\left[\frac{a+b}{a(1+w^2)+2aw+bw} \right]^6$$

$$= \left(\frac{a+b}{1aw+2aw+bw} \right)^6$$

$$= \left(\frac{a+b}{3aw+bw} \right)^6 = \left(\frac{a+b}{w(3a+b)} \right)^6$$

$$= \left(\frac{1}{w} \right)^6 = \left(\frac{w^3}{w} \right)^6 = (w^2)^6 = w^{12} = 1 = R-h-S$$

وزارة 2023 الدور الأول

$$x = \frac{5-2w^2}{5w-2}, y = \frac{3+4w}{3w^2+4} \quad \text{بـ إذا كن:$$

$$(x^2+y^2)=1 \quad \text{أثبت أن:}$$

$$x = \frac{5w^3-2w^2}{5w-2} = \frac{w^2(5w-2)}{(5w-2)} = (w^2)$$

$$y = \frac{3w^3+4w}{3w^2+4} = \frac{w(3w^2+4)}{(3w^2+4)} = (w)$$

$$h-h-S = [(w^2)^2 + (w)^2]$$

$$= (w^4 + w^2)$$

$$= (w+w^2)^2 = (-1)^2 = 1 = R-h-S$$

وزارة 2024 الدور الثاني

$$\left(\frac{1+i}{1-i} \right)^5 \left(\frac{5w^2-1}{5+w^2} \right)^{12} = i \quad \text{بـ أثبت أن:}$$

$$h-h-S = \left(\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} \right)^5 \left(\frac{5w^2+1}{5+w^2} \right)^{12}$$

$$= \left(\frac{(1+i+i-1)}{2} \right)^5 \left(\frac{w^2(5+w^2)}{(5+w^2)} \right)^{12}$$

$$= \left(\frac{2i}{2} \right)^5 (w^2 i)^{12}$$

$$= i^5 \cdot w^{24} i^{12}$$

$$= (i)(1)(1)$$

$$= i = R-h-S$$