

تدريب/ جد الصيغة القطبية :

① $Z = 27(1)$

$$Z = 27(\cos 0 + i \sin 0)$$

② $Z = 27(-1)$

$$Z = 27(\cos \pi + i \sin \pi)$$

③ $Z = 27(i)$

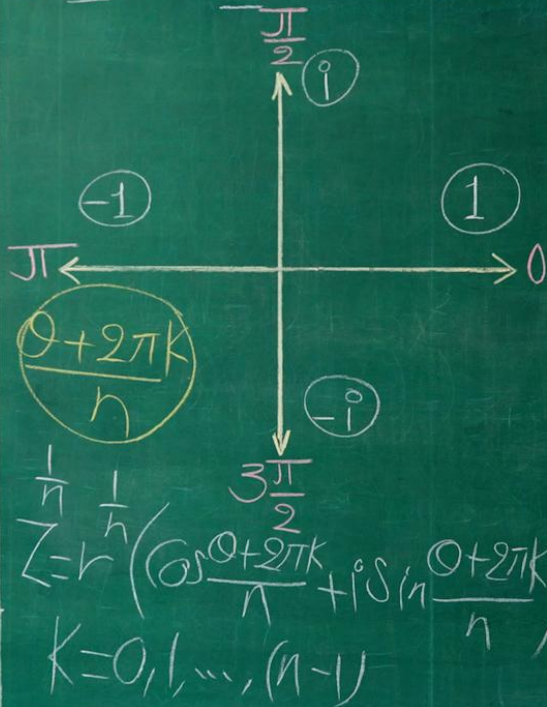
$$Z = 27\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

④ $Z = 27(i)$

$$Z = 27\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$$

② في حالة العدد المركب

$C = \underline{a}$ or $C = \underline{bi}$



الأعداد الصيغة القطبية

① في حالة العدد المركب

$C = a + bi$ كامل

① نضع العدد بالصيغة العادية $Z = x + yi$
ثم الديكارتيّة $Z(x, y)$

② نجد المقيار $r = \|Z\| = \sqrt{x^2 + y^2}$

③ نجد السعة:
 $\cos \theta = \frac{x}{r}$
 $\sin \theta = \frac{y}{r} \rightarrow \theta \Rightarrow \arg(Z)$

④ الصيغة القطبية
 $Z = r(\cos \theta + i \sin \theta)$

$$K=0 \Rightarrow Z_1 = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$$

$$Z_1 = 2(0 + i) = \boxed{2i}$$

$$K=1 \Rightarrow Z_2 = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$$

$$Z_2 = 2(-\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \boxed{-\sqrt{3} - i}$$

$$Z_2 = \boxed{-\sqrt{3} - i}$$

$$K=2 \Rightarrow Z_3 = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6})$$

$$Z_3 = 2(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \boxed{\sqrt{3} - i}$$

$$S = \{2i, -\sqrt{3} - i, \sqrt{3} - i\}$$

وزارة 2023 الدور الأول

من حل المعادلة بأستخدام
نتيجة مبرهنة دي موافر

$$\frac{2Z^3}{9} + 16 = 0 \quad i$$

$$2Z^3 + 16i = 0 \Rightarrow Z^3 = -\frac{16i}{2}$$

$$Z^3 = -8i = 8(-i)$$

$$Z = 8(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})^{\frac{1}{3}}$$

$$Z = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})^{\frac{1}{3}}$$

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$$Z = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})^{\frac{1}{3}}$$

$$K=0, 1, 2$$

$$K=0$$

$$Z_1 = 3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$Z_1 = 3(\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \boxed{\frac{3\sqrt{3}}{2} + \frac{3}{2}i}$$

$$K=1$$

$$Z_2 = 3(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$$

$$Z_2 = 3(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \boxed{-\frac{3\sqrt{3}}{2} + \frac{3}{2}i}$$

$$K=2$$

$$Z_3 = 3(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$$

$$Z_3 = 3(0 - i) = \boxed{-3i}$$

$$Z_3 = 3(0 - i) = \boxed{-3i}$$

$$Z_3 = 3(0 - i) = \boxed{-3i}$$

وزارة 2016 الدور الثاني

من بأستخدام نتيجة مبرهنة
دي موافر جذور التكعيبة

للمعادلة $(Z^3 - 27i)$

$$Z^3 = 27i$$

$$Z = 27(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})^{\frac{1}{3}}$$

$$Z = 3(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})^{\frac{1}{3}}$$

$$Z = 3(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})^{\frac{1}{3}}$$

$$Z = 3(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})^{\frac{1}{3}}$$

$$K=0, 1, 2$$

$$k=1 \Rightarrow Z_2 = 2(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$$

$$Z_2 = 2(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i) = -\sqrt{2} + \sqrt{2}i$$

$$k=2 \Rightarrow Z_3 = 2(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4})$$

$$Z_3 = 2(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i) = -\sqrt{2} - \sqrt{2}i$$

$$k=3 \Rightarrow Z_4 = 2(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$$

$$Z_4 = 2(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i) = \sqrt{2} - \sqrt{2}i$$

$$\sqrt{2} + \sqrt{2}i$$

$$-\sqrt{2} + \sqrt{2}i$$

$$-\sqrt{2} - \sqrt{2}i$$

$$\sqrt{2} - \sqrt{2}i = 0$$

وزارة 2015 دور الأول
7/ جذور الأربعة للعدد (-16)
بأستخدام نتيجة مبرهنة دي موافر

$$Z^4 = 16(-1)$$

$$Z^4 = 16(\cos \pi + i \sin \pi)$$

$$Z = 2(\cos \pi + i \sin \pi)^{\frac{1}{4}}$$

$$Z = 2(\cos \frac{\pi + 2\pi k}{4} + i \sin \frac{\pi + 2\pi k}{4})$$

$$k=0 \Rightarrow Z_1 = 2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$$

$$Z_1 = 2(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i) = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}i$$

$$Z_1 = \sqrt{2} + \sqrt{2}i$$

$$k=0 \Rightarrow x_1 = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$$

$$x_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$k=1 \Rightarrow x_2 = \cos \pi + i \sin \pi$$

$$x_2 = -1$$

$$k=2 \Rightarrow x_3 = \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}$$

$$x_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\delta = \left\{ \frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, \frac{1}{2} - \frac{\sqrt{3}}{2}i \right\}$$

وزارة 2019 الدور الثالث
6/ بأستخدام نتيجة مبرهنة
دي موافر حل المعادلة

$$x^3 + 1 = 0$$

$$x^3 = -1$$

$$x = \cos \pi + i \sin \pi$$

$$x = (\cos \pi + i \sin \pi)^{\frac{1}{3}}$$

$$x = \cos \frac{\pi + 2\pi k}{3} + i \sin \frac{\pi + 2\pi k}{3}$$

$$k=0, 1, 2$$