

بن / احسب يا سید احمد فقهہ در کوا قرا

$$L = \cos \frac{45}{12} \frac{\pi}{4} + i \sin \frac{45}{12} \frac{\pi}{4}$$

$$I = \cos 15 \frac{\pi}{4} + i \sin 15 \frac{\pi}{4}$$

$$Z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$$

① پرہیزہ و کواف ✓

② الاختصار

برهنه ديوان [ميفه عاديّه]

الأسر تركه

$$\underline{Z = x + yi} \Rightarrow (x, y)$$

$$r = \|Z\| = \sqrt{x^2 + y^2} \quad \text{نِجَالِ مَقْيَاس}$$

$$\theta = \arg(z)$$

قانون الحيف القلبية

$$Z = r(\cos \theta + i \sin \theta)$$

مركبة ديقا $z^n = r^n (\cos \theta + i \sin \theta)^n$

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

النوع الثاني: شکل الزاوية

n : أقل من ضعف $n\frac{\pi}{6}, n\frac{\pi}{4}, n\frac{\pi}{3}$

خبر ← ریح ← حذیف

النوع الثالث: شغل الزاوية

$n: \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{6}$ n عرض صفحه

فسيحة ← راقى

$$\begin{aligned}
 &= \frac{1}{2\sqrt{2}} (\cos 7\frac{\pi}{4} - i \sin 7\frac{\pi}{4}) \\
 &= \frac{1}{2\sqrt{2}} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) \\
 &= \frac{1}{2\sqrt{2}} (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i) \\
 &= \frac{1}{4} + \frac{1}{4} i
 \end{aligned}$$

وزارة في 2018 الأول

4 باستخدام مبرهنة دي موافر

$$\begin{aligned}
 &(-1-i)^3 = (-1-\sqrt{-1})^3 \\
 &Z = -1-i \Rightarrow (-1, -1) \\
 &r = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2} \\
 &\cos \theta = \frac{x}{r} = -\frac{1}{\sqrt{2}} \quad \theta = \frac{3\pi}{4} \\
 &\sin \theta = \frac{y}{r} = -\frac{1}{\sqrt{2}} \quad \text{الدع إلى أن} \\
 &\arg(Z) = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \\
 &Z = \sqrt{2} (\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}) \\
 &= \frac{1}{(\sqrt{2})^3} (\cos 15\frac{\pi}{4} - i \sin 15\frac{\pi}{4})
 \end{aligned}$$

$$\begin{aligned}
 &= 128\sqrt{2} (\cos 7\frac{\pi}{4} + i \sin 7\frac{\pi}{4}) \\
 &= 128\sqrt{2} (\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}) \\
 &= 128\sqrt{2} (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i) \\
 &= \frac{128\sqrt{2}}{\sqrt{2}} - \frac{128\sqrt{2}}{\sqrt{2}} i \\
 &= 128 - 128i
 \end{aligned}$$

في 2022

استخدام مبرهنة دي موافر

$$\begin{aligned}
 &(-2+2i)^5 \\
 &Z = -2+2i \quad (-2, 2) \\
 &r = \sqrt{x^2+y^2} = \sqrt{4+4} = \sqrt{8} \\
 &\cos \theta = \frac{x}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}} \\
 &\sin \theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \\
 &\arg(Z) = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \\
 &Z = \sqrt{8} (\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) \\
 &= 128\sqrt{2} (\cos 15\frac{\pi}{4} + i \sin 15\frac{\pi}{4})
 \end{aligned}$$

$$= \frac{1}{(4)^2} (\cos \frac{22\pi}{6} - i \sin \frac{22\pi}{6})$$

$$= \frac{1}{16} (\cos \frac{11\pi}{3} - i \sin \frac{11\pi}{3})$$

$$= \frac{1}{16} (\cos \frac{5\pi}{3} - i \sin \frac{5\pi}{3})$$

$$= \frac{1}{16} (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$= \frac{1}{16} (\frac{1}{2} + \frac{\sqrt{3}}{2} i)$$

$$= \frac{1}{32} + \frac{\sqrt{3}}{32} i$$

وزارة 2019 الدور الأول
6 باستخدام مبرهنة دي موافر
جد $(2\sqrt{3} - 2i)^2$

$$Z = 2\sqrt{3} - 2i \Rightarrow (2\sqrt{3}, -2)$$

$$r = \sqrt{x^2 + y^2} = \sqrt{12 + 4} = \sqrt{16} = 4$$

$$\cos \theta = \frac{x}{r} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \quad \theta = \frac{\pi}{6}$$

$$\sin \theta = \frac{y}{r} = \frac{-2}{4} = -\frac{1}{2} \quad \text{الربع الرابع}$$

$$\arg(z) = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$Z^2 = (4)^2 (\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6})$$

$$Z^4 = (9) (\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$$

$$= \frac{1}{(2)^4} (\cos \frac{20\pi}{3} - i \sin \frac{20\pi}{3})$$

$$= \frac{1}{16} (\cos \frac{4\pi}{3} - i \sin \frac{4\pi}{3})$$

$$= \frac{1}{16} (-\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})$$

$$= \frac{1}{16} (-\frac{1}{2} - \frac{\sqrt{3}}{2} i)$$

$$= -\frac{1}{32} - \frac{\sqrt{3}}{32} i$$

$$\frac{x^n}{y^n} = (\frac{x}{y})^n$$

وزارة 2018 دور الثاني
5 باستخدام مبرهنة دي موافر
أجب $\frac{1}{(1 - \sqrt{3}i)^4}$

$$(1 - \sqrt{3}i)^{-4}$$

$$Z = 1 - \sqrt{3}i \quad (1, -\sqrt{3})$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\cos \theta = \frac{x}{r} = \frac{1}{2} \quad \theta = \frac{\pi}{3}$$

$$\sin \theta = \frac{y}{r} = -\frac{\sqrt{3}}{2} \quad \text{الربع الثالث}$$

$$\arg(z) = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\arg(z) = 2\pi - \frac{\pi}{3} = \left(5\frac{\pi}{3}\right)$$

$$Z = (2)^{-7} (\cos 5\frac{\pi}{3} + i \sin 5\frac{\pi}{3})$$

$$= \frac{1}{(2)^7} (\cos 35\frac{\pi}{3} - i \sin 35\frac{\pi}{3})$$

$$= \frac{1}{128} (\cos 5\frac{\pi}{3} - i \sin 5\frac{\pi}{3})$$

$$= \frac{1}{128} (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$= \frac{1}{128} \left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{256} + i \frac{\sqrt{3}}{256}$$

وزارة 2023 الدور الاول، الدراسة الزمائية

$$Z = \frac{7 + \sqrt{3}i}{1 + 2\sqrt{3}i}$$

بأستخدام مبرهنة دجواقر

$$Z = \frac{7 + \sqrt{3}i \cdot 1 - 2\sqrt{3}i}{1 + 2\sqrt{3}i \cdot 1 - 2\sqrt{3}i}$$

$$Z = \frac{7 - 14\sqrt{3} + \sqrt{3}i + 2(3)}{1 + 12}$$

$$Z = \frac{7 - 13\sqrt{3}i + 6}{13} = \frac{13}{13} - \frac{13\sqrt{3}i}{13}$$

$$Z = 1 - \sqrt{3}i \quad \left(\begin{matrix} x & y \\ 1 & -\sqrt{3} \end{matrix}\right)$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\cos \theta = \frac{x}{r} = \frac{1}{2} \quad \left\{ \begin{matrix} \theta = \frac{\pi}{3} \\ \sin \theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} \end{matrix} \right.$$

الربع الرابع

وزارة 2014 الدور الثاني

$$Z = (2)^{-9} (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$= \frac{1}{(2)^9} (\cos 9\frac{\pi}{6} - i \sin 9\frac{\pi}{6})$$

$$= \frac{1}{512} (\cos 3\frac{\pi}{2} - i \sin 3\frac{\pi}{2})$$

$$= \frac{1}{512} (0 + i)$$

$$= 0 + \frac{1}{512} i$$

بأستخدام مبرهنة دجواقر

$$(\sqrt{3} + i)^{-9}$$

$$Z = \sqrt{3} + i \Rightarrow (\sqrt{3}, 1)$$

$$r = \|Z\| = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$\cos \theta = \frac{x}{r} = \frac{\sqrt{3}}{2} \quad \left\{ \begin{matrix} \theta = \frac{\pi}{6} \\ \sin \theta = \frac{y}{r} = \frac{1}{2} \end{matrix} \right.$$

$$\arg(z) = \left(\frac{\pi}{6}\right)$$

وزارة 2021 الدور الثاني

بسط: $(\sin \frac{\pi}{6} + i \cos \frac{\pi}{3})^{-5}$

$(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})^{-5}$

دعنا نأخذ $= \cos \frac{\pi}{3} - i \sin \frac{\pi}{3}$

$= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} i$

θ	$\sin \theta$	$\cos \theta$
$30^\circ \quad \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$60^\circ \quad \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$45^\circ \quad \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

$\boxed{\sin \frac{\pi}{6} = \cos \frac{\pi}{3}}$

$\cos \frac{\pi}{6} = \sin \frac{\pi}{3}$

اثرائي ... $2(\sin \frac{\pi}{6} - i \cos \frac{\pi}{6})^{10}$ مبسط

$= 2(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6})^{10}$

$= 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})^{10}$ $10 \cdot \frac{\pi}{3}$ $10 \div 6 = 1$ اباقي 4

$= 2(\cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3})$

$= 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$ $4 \cdot \frac{\pi}{3}$ $4(60) = 240^\circ$ ثالث

$= 2(-\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})$

$= 2(-\frac{1}{2} - \frac{\sqrt{3}}{2} i)$

$= -\frac{2}{2} - \frac{2\sqrt{3}}{2} i = -1 - \sqrt{3} i$