

موضوع: المصفات القطبية

① يجب أن يكون العدد بالمصفاة العادية
 $Z = x + yi$ ثم المصفاة الديكارتية $Z(x, y)$

② نجد المقدار $r = \|Z\| = \sqrt{x^2 + y^2}$
 $\text{mod}(z)$

③ نجد السمت: $\left[\begin{array}{l} \cos \theta = \frac{x}{r} \\ \sin \theta = \frac{y}{r} \end{array} \right] \rightarrow \theta \Rightarrow \arg(z)$

④ قانون المصفاة القطبية
 $Z = r(\cos \theta + i \sin \theta)$

$$Z = r(\cos \theta + i \sin \theta)$$

$\arg(z)$

$$\textcircled{3} Z = (1 + \sqrt{3}i)^2$$

$$Z = 1 + 2\sqrt{3}i - 3 \Rightarrow Z = -2 + 2\sqrt{3}i$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

$$\cos \theta = \frac{x}{r} = \frac{-2}{4} = -\frac{1}{2} \quad \left. \begin{array}{l} \cos \theta = \frac{x}{r} = \frac{-2}{4} = -\frac{1}{2} \\ \sin \theta = \frac{y}{r} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \end{array} \right\} \theta = \frac{\pi}{3}$$

$$\sin \theta = \frac{y}{r} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\arg(z) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$Z = 4 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$\textcircled{2} Z = 5 - 5i \quad \left(\overset{x}{5}, \overset{y}{-5} \right)$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

$$\cos \theta = \frac{x}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \left. \begin{array}{l} \cos \theta = \frac{x}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \\ \sin \theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} \end{array} \right\} \theta = \frac{\pi}{4}$$

$$\sin \theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\arg(z) = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$Z = 5\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

مس/عبر عن كحل من الامداد بالصيغة القطبية:

$$\textcircled{1} Z = 3 - 3\sqrt{3}i \quad \left(\overset{x}{3}, \overset{y}{-3\sqrt{3}} \right)$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{9 + 27} = \sqrt{36} = 6$$

$$\cos \theta = \frac{x}{r} = \frac{3}{6} = \frac{1}{2} \quad \left. \begin{array}{l} \cos \theta = \frac{x}{r} = \frac{3}{6} = \frac{1}{2} \\ \sin \theta = \frac{y}{r} = \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2} \end{array} \right\} \theta = \frac{\pi}{3}$$

$$\sin \theta = \frac{y}{r} = \frac{-3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$$

$$\arg(z) = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$Z = 6 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$$

من/عبر عن كل من الامداد بالصيغة القطبية:

$$④ Z = \frac{4}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{4(1+\sqrt{3}i)}{4}$$

$$\bar{Z} = 1+\sqrt{3}i \quad (1, \sqrt{3})$$

$$r = \|Z\| = \sqrt{x^2 + y^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\cos \theta = \frac{x}{r} = \frac{1}{2} \quad \theta = \frac{\pi}{3}$$

$$\sin \theta = \frac{y}{r} = \frac{\sqrt{3}}{2} \quad \text{الربع الأول}$$

$$\arg(z) = \left(\frac{\pi}{3}\right)$$

$$Z = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

وزارة 2003 عدد 2

من/ اذا كان Z عدد مركب مقياسه (3)

وسمته $\left(\frac{\pi}{3}\right)$ جد محل من الشكل الجبري والديكارتي.

$$r = 3 \quad \arg(z) = \left(\frac{\pi}{3}\right)$$

$$\bar{Z} = 3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$Z = 3 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$$

$$Z = \frac{3}{2} + \frac{\sqrt{3}}{2} i \quad \text{الصيغة العادية (الكبرية)}$$

$$Z = \left(\frac{3}{2}, \frac{\sqrt{3}}{2} \right) \quad \text{الديكارتية}$$

أضافي

من/ اذا كان Z عدد مركب مقياسه $(\sqrt{2})$

وسمته $\left(\frac{\pi}{4}\right)$ جد شكله الجبري والديكارتي.

$$r = \sqrt{2} \quad \arg(z) = \left(\frac{\pi}{4}\right)$$

$$Z = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$Z = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right)$$

$$Z = \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} i$$

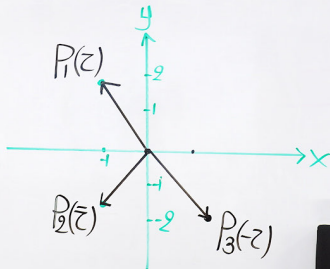
$$Z = 1 + i \quad \text{الكبرية}$$

$$Z = (1, 1) \quad \text{الديكارتية}$$

$$Z = -1 + \sqrt{3}i \quad P_1(Z) = (-1, \sqrt{3})$$

$$\bar{Z} = -1 - \sqrt{3}i \quad P_2(\bar{Z}) = (-1, -\sqrt{3})$$

$$-Z = 1 - \sqrt{3}i \quad P_3(-Z) = (1, -\sqrt{3})$$



فزارى 2025 العدد الأول

س/ ليكن $Z = -1 + \sqrt{3}i$ جد الصيغ

القطبية للعدد Z ثم قتل على شكل

أوجد $Z, \bar{Z}, -Z$

$$Z = -1 + \sqrt{3}i \Rightarrow (-1, \sqrt{3})$$

$$r = \|Z\| = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\cos \theta = \frac{x}{r} = -\frac{1}{2}$$

$$\sin \theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$$

$$\left. \begin{array}{l} \cos \theta = -\frac{1}{2} \\ \sin \theta = \frac{\sqrt{3}}{2} \end{array} \right\} \theta = \frac{\pi}{3} \text{ الربع الثاني}$$

$$\arg(z) = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$Z = 2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

وزارة 2006 الدور الأول

مع إذا كان Z عدداً مركباً بقياسه (4) وسقطه

$(5\frac{\pi}{6})$ جد شكله الجبري والديكارتي.

$$(r=4) \quad \arg(z)=5\frac{\pi}{6}$$

$$Z = 4 (\cos 5\frac{\pi}{6} + i \sin 5\frac{\pi}{6})$$

$$Z = 4 (-\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$Z = 4 (-\frac{\sqrt{3}}{2} + \frac{1}{2}i)$$

$$Z = -\frac{4\sqrt{3}}{2} + \frac{4}{2}i$$

$$Z = -2\sqrt{3} + 2i \quad \text{الجبرية}$$

$$Z = (-2\sqrt{3}, 2) \quad \text{الديكارتي}$$

$$\text{نقطة } 5\frac{\pi}{6}$$

$$5(30^\circ) = 150^\circ$$

الدع الثاني

$$\cos \theta \quad \sin \theta$$

وزارة 2099 الدور الأول

مع يكون المحادلة التربيعية ذات المعاملات الحقيقية وأحد

جذريها العدد المركب الذي بقياسه (2) وسقطه $(5\frac{\pi}{3})$.

$$m = 1 - \sqrt{3}i, \quad L = 1 + \sqrt{3}i \quad m = 2 (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$m + L = (1 - \sqrt{3}i) + (1 + \sqrt{3}i) = 2 \quad m = 2 (\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})$$

$$m \cdot L = (1 - \sqrt{3}i)(1 + \sqrt{3}i) = 4 \quad m = 2 (\frac{1}{2} - \frac{\sqrt{3}}{2}i)$$

$$x^2 - (m+L)x + (m \cdot L) = 0 \quad m = \frac{p}{2} - \frac{q\sqrt{3}}{2}i$$

$$x^2 - 2x + 4 = 0 \quad m = 1 - \sqrt{3}i$$

المعادلة التربيعية

① $(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ ② $\frac{\pi}{3}$ (نوع 1)

$= -\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ $2 \frac{\pi}{3}$ (2)

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ $2(60^\circ) = 120^\circ$

② $(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$ $7 \frac{\pi}{6}$ (نوع 2)

$= -\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}$ $7(30^\circ) = 210^\circ$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$ $\frac{\pi}{6}$ (نوع 3)

③ $(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$ $7 \frac{\pi}{4}$ (نوع 4)

$= \cos \frac{\pi}{4} - i \sin \frac{\pi}{4}$ $7(45^\circ) = 315^\circ$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ $\frac{\pi}{4}$ (نوع 5)

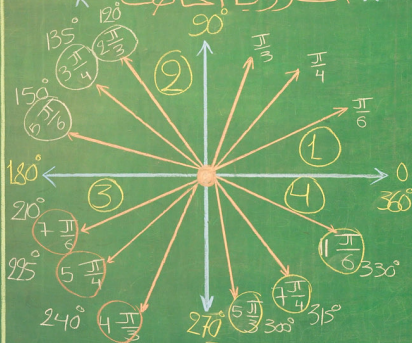
④ $(\cos \frac{35\pi}{4} + i \sin \frac{35\pi}{4})$ $35 \frac{\pi}{4}$ (نوع 6)

$= (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$ $\frac{\pi}{4}$ (نوع 7)

$= \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ $\frac{\pi}{4}$ (نوع 8)

$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$ $\frac{\pi}{3}$ (نوع 9)

* الزوايا الخاصة *

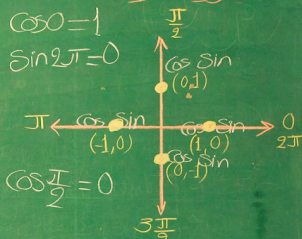


النوع ②: n: أقل من ضعف المقام
النوع ③: n: أكبر من ضعف المقام

خرب ← ربع ← حرف
قسم ← باقي

$35 \div 8 = 4$

* الزوايا المحورية *



① $(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) = 0 + i$

② $(\cos \pi + i \sin \pi) = -1 + 0i$

③ $(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}) = 0 - i$

④ $(\cos 2\pi + i \sin 2\pi) = 1 + 0i$

وزارة 2020 دور الثاني

س/ اذا كان $Z = -2 + bi$ عدداً مركباً القيمة الأساسية

لسقطه $(4\frac{\pi}{3})$ جدي $b \in \mathbb{R}$

$$Z = -2 + bi \Rightarrow (-2, b)$$

نقطة $(4\frac{\pi}{3})$

$4(60^\circ) = 240^\circ$
الربع الثالث

$$\cos \theta = \frac{x}{r}$$

$$\cos 4\frac{\pi}{3} = \frac{-2}{r}$$

$$\cos \frac{\pi}{3} = \frac{-2}{r}$$

$$-\frac{1}{2} = \frac{-2}{r}$$

$$(r = 4)$$

$$\sin \theta = \frac{y}{r}$$

$$\sin 4\frac{\pi}{3} = \frac{b}{r}$$

$$\sin \frac{\pi}{3} = \frac{b}{r}$$

$$-\frac{\sqrt{3}}{2} = \frac{b}{4}$$

$$\frac{b}{4} = -\frac{\sqrt{3}}{2} \Rightarrow b = -2\sqrt{3}$$

سؤال التلفزيون التربوي

س/ اذا كان $Z = m + 1 - \sqrt{3}i$ عدداً مركباً القيمة

الأساسية لسقطه $(4\frac{\pi}{3})$ جدي $m \in \mathbb{R}$

$$Z = m + 1 - \sqrt{3}i \Rightarrow (m+1, -\sqrt{3})$$

$$\cos \theta = \frac{x}{r}$$

$$\cos 4\frac{\pi}{3} = \frac{m+1}{r}$$

$$\cos \frac{\pi}{3} = \frac{m+1}{r}$$

$$-\frac{1}{2} = \frac{m+1}{2}$$

$$2m+2 = -2$$

$$2m = -2-2 \Rightarrow m = -2$$

$$(m = -2)$$

$$\sin \theta = \frac{y}{r}$$

$$\sin 4\frac{\pi}{3} = \frac{-\sqrt{3}}{r}$$

$$\sin \frac{\pi}{3} = \frac{-\sqrt{3}}{r}$$

$$-\frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{r}$$

$$\frac{\sqrt{3}r}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}} \Rightarrow r = 2$$